attain to the same certainty as laws, but this result must follow on an examination of the nature of the propositions; for since laws are also not wholly free from the possibility of error, the difference can never serve to distinguish them from laws.

Closely connected with "theory" is another word, "hypothesis". In fact the two terms are often regarded, especially in the older literature, as synonymous; Laplace's Nebular Theory and Nebular Hypothesis are used indifferently. An hypothesis is, strictly speaking, a proposition which is put forward for consideration, and concerning the truth or falsity of which nothing is asserted until the consideration is completed. It is thus necessarily associated with doubt, but with doubt of a negative rather than of a positive kind, with the doubt which consists of a suspense of judgement rather than with the doubt which consists of an inclination to disbelieve. In current usage, however, the word, especially in the adjectival form, almost always connotes doubt of the second kind; to term a view hypothetical is practically equivalent to expressing dissent from it. From this connotation I want also to be free. The word will be given a special sense which is justified by its origin to this extent than an hypothesis will always be a proposition which cannot be judged to be either true or false unless there are added to it certain other propositions, although it has a distinct significance apart from these other propositions. Hypothesis and hypothetical must be taken to imply doubt of the first kind and never doubt of the second.

What I do mean by a theory. I have now stated what I do not mean by a theory and an hypothesis; it remains to state what I do mean.

A theory is a connected set of propositions which are divided into two groups. One group consists of statements about some collection of ideas which are characteristic of the theory; the other group consists of statements of the relation between these ideas and some other ideas of a different nature. The first group will be termed collectively the "hypothesis" of the theory; the second group the "dictionary". The hypothesis is so called, in accordance with the sense that has just been stated, because the propositions composing it are incapable of proof or of disproof by themselves; they must be significant, but, taken apart from the dictionary, they appear arbitrary assumptions. They may be considered accordingly as providing a "definition by postulate" of the ideas which are characteristic of the hypothesis. The ideas which are related by means of the dictionary to the ideas of the hypothesis are, on the other hand, such that something is known about them apart from the theory. It must be possible to determine, apart from all knowledge of the theory, whether certain propositions involving these ideas are true or false. The dictionary relates some of these propositions, of which the truth or falsity is known, to certain propositions involving the hypothetical ideas by stating that if the first set of propositions is true then the second set is true and vice versa; this relation may be expressed by the statement that the first set implies the second.

In scientific theories (for it seems that there may be sets of propositions

having exactly the same features in departments of knowledge other than science) the ideas connected by means of the dictionary to the hypothetical ideas are always concepts in the sense of Chapter II, that is collections of fundamental judgements related in laws by uniform association; and the propositions involving these ideas, of which the truth or falsity is known, are always laws. Accordingly those ideas involved in a theory which are not hypothetical ideas will be termed concepts; it must be remembered that this term is used in a very special sense; concepts depend for their validity on laws, and any proposition in which concepts are related to concepts is again a law. Whether there is any necessary limitation on the nature of the ideas which can be admitted as hypothetical ideas is a question which requires much consideration; but one limitation is obviously imposed at the outset by the proviso that propositions concerning them are arbitrary, namely that they must not be concepts. As a matter of fact the hypothetical ideas of most of the important theories of physics, but not of other sciences, are mathematical constants and variables. (Except when the distinction is important, the term "variable" will be used in this chapter to include constants.)

The theory is said to be true if propositions concerning the hypothetical ideas, deduced from the hypothesis, are found, according to the dictionary, to imply propositions concerning the concepts which are true, that is to imply laws; for all true propositions concerning concepts are laws. And the theory is said to explain certain laws if it is these laws which are implied by the propositions concerning the hypothetical ideas.

An illustration will make the matter clearer. To spare the feelings of the scientific reader and to save myself from his indignation, I will explain at the outset that the example is wholly fantastic, and that a theory of this nature would not be of the slightest importance in science. But when it has been considered we shall be in a better position to understand why it is so utterly unimportant, and in what respects it differs from valuable scientific theories.

The hypothesis consists of the following mathematical propositions:

(1) $u, v, w, \ldots$ are independent variables.

(2) a is a constant for all values of these variables.

(3) b is a constant for all values of these variables.

(4) $c = d$, where c and d are dependent variables.

The dictionary consists of the following propositions:

(1) The assertion that $(c^2 + d^2)\, a = R$, where R is a positive and rational number, implies the assertion that the resistance of some definite piece of pure metal is R.

(2) The assertion that $cd/b = T$ implies the assertion that the temperature of the same piece of pure metal is T.

From the hypothesis we deduce

$$(c^2 + d^2)\, a \Big/ \frac{cd}{b} = 2ab = \text{constant}.$$

Interpreting this proposition by means of the dictionary we arrive at the following law:

The ratio of the resistance of a piece of pure metal to its absolute temperature is constant.

This proposition is a true law (or for our purpose may be taken as such). The theory is therefore true and explains the law.

This example, absurd though it may seem, will serve to illustrate some of the features which are of importance in actual theories. In the first place, we may observe the nature of the propositions, involving respectively the hypothetical ideas and the concepts, which are stated by the dictionary to imply each other. When the hypothetical ideas are mathematical variables, the concepts are measurable concepts (an idea of which much will be said hereafter), and the propositions related by mutual implication connect the variables, or some function of them, to the same number as these measurable concepts. When such a relation is stated by the dictionary it will be said for brevity that the function of the hypothetical ideas "is" the measurable concept; thus, we shall say that $(c^2 + d^2)\, a$ and cd/b "are" respectively the resistance and temperature. But it must be insisted that this nomenclature is adopted only for brevity; it is not meant that in any other sense of that extremely versatile word "is" $(c^2 + d^2)\, a$ is the resistance; for there are some senses of that word in which a function of variables can no more "be" a measurable concept than a railway engine can "be" the year represented by the same number.

If an hypothetical idea is directly stated by the dictionary to be some measurable concept, that idea is completely determined and every proposition about its value can be tested by experiment. But in the example which has been taken this condition is not fulfilled. It is only functions of the hypothetical ideas which are measurable concepts. Moreover since only two functions, which involve four mathematical variables and between which one relation is stated by the hypothesis, are stated to be measurable concepts, it is impossible by a determination of those concepts to assign definitely numerical values to them. If some third function of them had been stated to be some third measurable concept, then it would have been possible to assign to all of them numerical values in an unique manner. If further some fourth function has been similarly involved in the dictionary, the question would have arisen whether the values determined from one set of three functions is consistent with those determined from another set of three.

These distinctions are important. There is obviously a great difference between a theory in which some proposition based on experiment can be asserted about each of the hypothetical ideas, and one in which nothing can be said about these ideas separately, but only about combinations of them. There is also a difference between those in which several statements about those ideas can be definitely shown to be consistent and those in which such statements are merely known not to be inconsistent. In these respects actual theories differ in almost all possible degrees; it very often happens that some of the hypothetical ideas can be directly determined by experiment while others cannot; and in such cases there is an important difference between the two classes of ideas. Those which can be directly determined are often confused with the concepts to which they are directly related, while those which cannot are recognised as distinctly theoretical. But it must be noticed that a distinction of this nature has no foundation. The ideas of the hypothesis are never actually concepts; they are related to concepts only by means of the dictionary. Whatever the nature of the dictionary, all theories have this in common that no proposition based on experimental evidence can be asserted concerning the hypothetical ideas except on the assumption that the propositions of the theory are true. This is a most important matter which must be carefully borne in mind in all our discussions.

It will be observed that in our example there are no propositions in the dictionary relating any of the independent variables of the hypothesis to measurable concepts. This feature is characteristic of such theories. The nature of the connection between the independent variables and the concepts is clear from the use made, in the deduction of the laws, of the fact that a and b are constants, not varying with the independent variables. The conclusion that the electrical resistance is proportional to the absolute temperature would not follow unless $(c^2 + d^2)\, a$ were the resistance in the same state of the system the same as that in which cd/b is the temperature; and on the other hand it would not follow if a and b were not the same constants in all the propositions of the dictionary. Accordingly the assertion that a or b is a constant must imply that it is the same so long as the state of the system to which the concepts refer is the same; the independent variables on the contrary may change without a corresponding change in the state of the system. If therefore there is to be in the dictionary a proposition introducing the independent variables, it must state that a change in the independent variables does *not* imply a change in the state of the system; the omission of these variables from the dictionary must be taken to mean a definite negative statement. On the other hand, the independent variables may bear some relation to measurable concepts, so long as these concepts are not properties of the system. Thus, in almost all theories of this type, one of the independent variables is called the "time", and the use of this name indicates that it is related in some manner to the physically measurable "time" since some agreed datum. What exactly is this relation we shall have to inquire in the third part of this volume, but it is to be noted that a relation between one of the independent variables and physically measured time is not inconsistent with the statement that a change in this variable does not imply any change in the state of the system; for it is one of the essential properties of a system that its state should be, in a certain degree and within certain limits, independent of the time.

In some theories again, there are dependent variables which are not mentioned in the dictionary. But in such cases the absence of mention is not to be taken as involving the definite assertion that there is no relation between these variables and the concepts. It must always be regarded as possible that a further development of the theory may lead to their introduction into the dictionary.

An example of physical theories. The fantastic example on which this discussion has been based was introduced in order that, in defining a theory and examining some features of its formal constitution, we might be free from associated ideas which would be sure to arise if the example were taken from any actual theory. It is easier thus to realise the difference between the hypothesis of the theory and the dictionary, and between the nature of the ideas which are characteristic of those two parts of the theory, or to recognise that numerical values can be attributed by experiment to the hypothetical ideas only in virtue of the propositions of the theory. But now we have to consider whether there are any actual scientific propositions which have this formal constitution, and, if there are, whether the application of the term theory to them accords with the usual practice; further we have to decide, if we answer these questions in the affirmative, what it is that gives them a value so very much greater than that of the absurd example which has been used so far. For this purpose an actual scientific proposition will be taken which is generally considered to have considerable value and is always called a theory; and it will be shown that it has the formal constitution which has just been explained. It will thus appear that in one instance at least our definition accords with ordinary usage.

The theory which will be selected is the dynamical theory of gases. We shall start with it in its very simplest form, in which it explains only the laws of Boyle and Gay-Lussac. For such explanation no account need be taken of collisions between the molecules, which may therefore be supposed to be of infinitely small size. Though the theory in this form is known now not to be true, it will be admitted that it is as much a theory in this form as in its more complex modern form. By starting with the simplest form we shall abbreviate our original discussion and at the same time permit the interesting process of the development of a theory to be traced. And when the development of the theory is mentioned, it should be explained that the development traced will not be that which has actually occurred but that which might have occurred; no attention is paid to merely historical considerations. One further word of warning should be given at the outset. Objections have at times been raised to this theory, and to all of similar type, by those who would admit theories of a somewhat different nature. By taking the dynamical theory of Gases as an example I am not overlooking these objections or assuming in any way that all scientific theories are essentially the same in nature as the example; we shall discuss these matters later.

Let us then attempt to express the theory in the form which has been explained. The hypothesis of the theory may be stated as follows:

(1) There is a single independent variable t.

(2) There are three constants, m, v, and l, independent of t.

(3) There are $3n$ dependent variables (x_s, y_s, z_s) $(s = 1 \text{ to } n)$ which are continuous functions of t. They form a continuous three-dimensional series and are such that $(x_s^2 + y_s^2 + z_s^2)$ is invariant for all linear transformations of the type $x' = ax + by + cz$. (This last sentence is merely a way of

$F = f = \frac{\Delta}{\Delta t} m \frac{dx}{dt}$ $p = \frac{F}{s} = \frac{ma}{s}$

 $\frac{\Delta}{\Delta t}$

saying that (x, y, z) are related like rectangular coordinates; but since any definitely spatial notions might give the idea that the properties of the (x, y, z) were somehow determined by experiment, they have been avoided.)

(4) $\frac{d}{dt}(x_s, y_s, z_s)$ is constant, except when (x_s, y_s, z_s) is 0 or l; when it attains either of these values it changes sign.

(5) $\frac{1}{n}\sum_1^n \left(\frac{dx_s}{dt}\right)^2 = v^2$, and similar propositions for y_s and z_s.

The dictionary contains the following propositions:

(1) l is the length of the side of a cubical vessel in which a "perfect" gas is contained.

(2) nm is the mass of the gas, M.

(3) $\frac{1}{a} mv^2$ is T, the absolute temperature of the gas, where a is some number which will vary with the arbitrary choice of the degree of temperature[1].

(4) Let $\Delta m \frac{dx_s}{dt}$ be the change in $m \frac{dx_s}{dt}$ which occurs when x_s attains the value l; let $\Sigma_\gamma \Delta m \frac{dx_s}{dt}$ be the sum of all values of $\Delta m \frac{dx_s}{dt}$ for which t lies between t and $t + \gamma$; let

$$(p_a, p_b, p_c) = \mathrm{Lt}_{n\to\infty,\gamma\to\infty} \sum_{s=1}^{s=n} \frac{1}{\gamma} \Sigma_\gamma \Delta m \frac{d(x_s, y_s, z_s)}{dt},$$

then p_a, p_b, p_c are the pressures P_a, P_b, P_c on three mutually perpendicular walls of the cubical containing vessel.

From the propositions of the hypothesis it is possible to prove that

$$p_a = p_b = p_c = \frac{1}{3l^3} nmv^2.$$

But l^3 is V, the volume of the gas. If we interpret this proposition according to the dictionary we find

$$P_a = P_b = P_c = \frac{T}{V} \cdot \frac{an}{3},$$

which is the expression of Boyle's and Gay-Lussac's Laws, since $\frac{an}{3}$ is constant.

[1] The occurrence of a needs some remark. Is it a hypothetical idea or a measurable concept? It is neither. We shall consider its nature when we deal with temperature, but it may be stated here briefly why a number of this kind occurs in this entry in the dictionary and not in the others. The reason is this. Experiment shows that pv is proportional to T. For various reasons, which we shall discuss, we desire that the factor of proportionality shall *not* change if the unit of mass or the unit of pressure is changed; but we do not object to its changing when the degree of temperature changes. If we gave the factor a definite value once and for all, the degree of temperature would have to change when the units of mass and pressure changed; we wish to avoid this necessity and do so by changing the value of a when we change the degree. The value of a is therefore as purely arbitrary as the choice of a unit in any system of measurement.

The theory is here expressed in a form exactly similar to that of our original example, and it will now be seen that this form is not wholly artificial, but has a real significance. In explaining the laws by the theory, we do actually deduce propositions from the hypothesis and interpret them in experimental terms by means of the dictionary. Moreover the distinction between the various kinds of variable in respect of their connection with measurable concepts is apparent. l is directly connected by the dictionary to a measurable concept, and the attribution to it of a numerical value requires nothing but a knowledge of the dictionary; the hypothesis is not involved. At the other extreme, the variables or constants n, m, x_s, y_s, z_s cannot be given numerical values by experiment even with help of the hypothesis; only functions of these variables and not the variables separately can be determined. Between these two extremes lies the constant v. We have deduced from the hypothesis that $v^2 = \frac{3l^3 p_a}{nm}$. The right-hand side of this equation can be given by experiment a numerical value, namely $3\frac{VP_a}{M}$, by means of the dictionary, so that v can also be evaluated. But this evaluation depends wholly on the acceptance of the propositions of the hypothesis; apart from those propositions a statement that v has a certain numerical value does not assert anything which can be proved by experiment.

Having thus shown that the dynamical theory of gases is a theory in our sense, we must now ask what is the difference between this valuable theory and the trivial example with which we began? It lies, of course, in the fact that the propositions of the hypothesis of the dynamical theory of gases display an analogy which the corresponding propositions of the other theory do not display. The propositions of the hypothesis are very similar in form to the laws which would describe the motion of a large number of infinitely small and highly elastic bodies contained in a cubical box. If we had such a number of particles, each of mass m, occupying points in a box of side l represented by the coordinates (x_s, y_s, z_s), and initially in motion, then their momentum would change sign at each impact on the walls of the box. $l^2 (p_a, p_b, p_c)$ would be the rate of change of momentum at the walls of the box and would, accordingly, be the average force exerted upon those walls. And so on; it is unnecessary to state the analogy down to its smallest details. All these symbols, $m, l, t, x, y, z, \ldots$ would denote the numerical values of actually measurable physical concepts, and it would be a law that they were related in the way described; if they were actually measured and the resulting numerical values inserted in the equations stated those equations would be satisfied.

Further the propositions of the dictionary are suggested by the analogy displayed by the propositions of the hypothesis. p is called the "pressure", and the pressure of the gas P is specially related to the variable p, because p, in the law to which the hypothesis is analogous, would be the average pressure on the walls of the box actually observed. Similar considerations suggested

2 P

the establishment of the relation between nm and the total mass of the gas, and between l^3 and its volume. The basis of the relation established between T and mv^2 is rather more complex, and its full consideration must be left till we deal in detail with the theory as a part of actual physics; but again it lies in an analogy. Speaking roughly, we may say that the relation is made because, in the law of the elastic particles, mv^2 would be a magnitude which would be found to remain constant so long as the box containing the particles was isolated from all exterior interference, while in the case of the gas the temperature is found so to remain constant during complete isolation.

The importance of the analogy. We see then that the class of physical theories of which the theory of gases is a type has two characteristics. First they are of the form which has been described, consisting of an hypothesis and a dictionary; if they are to be true, they must be such that laws which are actually found to be true by observation can be deduced from the hypothesis by means of logical reasoning combined with translation through the dictionary. But in order that a theory may be valuable it must have a second characteristic; it must display an analogy. The propositions of the hypothesis must be analogous to some known laws.

This manner of expressing the formal constitution of a theory is probably not familiar to most readers, but there is nothing new in the suggestion that analogy with laws plays an important part in the development of theories. No systematic writer on the principles of science is in the least inclined to overlook the intimate connection between analogy and theories or hypotheses. Nevertheless it seems to me that most of them have seriously misunderstood the position. They speak of analogies as "aids" to the formations of hypotheses (by which they usually mean what I have termed theories) and to the general progress of science. But in the view which is urged here analogies are not "aids" to the establishment of theories; they are an utterly essential part of theories, without which theories would be completely valueless and unworthy of the name. It is often suggested that the analogy leads to the formulation of the theory, but that once the theory is formulated the analogy has served its purpose and may be removed and forgotten. Such a suggestion is absolutely false and perniciously misleading. If physical science were a purely logical science, if its object were to establish a set of propositions all true and all logically connected but characterised by no other feature, then possibly this view might be correct. Once the theory was established and shown to lead by purely logical deduction to the laws to be explained, then certainly the analogy might be abandoned as having no further significance. But, if this were true, there would never have been any need for the analogy to be introduced. Any fool can invent a logically satisfactory theory to explain any law. There is as a matter of fact no satisfactory physical theory which explains the variation of the resistance of a metal with the temperature. It took me about a quarter of an hour to elaborate the theory given on p. 123; and yet it is, I maintain, formally as satisfactory as any theory in physics. If nothing but this were required we should never lack theories to explain our

「analogy 없으면 이론은 무의미하다」

laws; a schoolboy in a day's work could solve the problems at which generations have laboured in vain by the most trivial process of trial and error. What is wrong with the theory of p. 123, what makes it absurd and unworthy of a single moment's consideration, is that it does not display any analogy; it is just because an analogy has not been used in its development that it is so completely valueless.

Analogy, so far from being a help to the establishment of theories, is the greatest hindrance. It is never difficult to find a theory which will explain the laws logically; what is difficult is to find one which will explain them logically and at the same time display the requisite analogy. Nor is it true that, once the theory is developed, the analogy becomes unimportant. If it were found that the analogy was false it would at once lose its value; if it were presented to someone unable to appreciate it, for him the theory would have little value. To regard analogy as an aid to the invention of theories is as absurd as to regard melody as an aid to the composition of sonatas. If the satisfaction of the laws of harmony and the formal principles of development were all that were required of music, we could all be great composers; it is the absence of the melodic sense which prevents us all attaining musical eminence by the simple process of purchasing a text-book.

The reason why the perverse view that analogies are merely an incidental help to the discovery of theories has ever gained credence lies, I believe, in a false opinion as to the nature of theories. I said just now that it was a commonplace that analogies were important in the framing of hypotheses, and that the name "hypotheses" was usually given in this connection to the propositions (or sets of propositions) which are here termed theories. This statement is perfectly true, but it is not generally recognised by such writers that the "hypotheses" of which they speak are a distinct class of propositions, and especially that they are wholly different from the class of laws; there is a tendency to regard an "hypothesis" merely as a law of which full proof is not yet forthcoming.

If this view were correct, it might be true that the analogy was a mere auxiliary to the discovery of laws and of little further use when the law was discovered. For once the law had been proposed the method of ascertaining whether or no it were true would depend in no way on the analogy; if the "hypothesis" were a law, its truth would be tested like that of any other law by examining whether the observations asserted to be connected by the relation of uniformity were or were not so connected. According as the test succeeded or failed, the law would be judged true or false; the analogy would have nothing to do with the matter. If the test succeeded, the law would remain true, even if it subsequently appeared that the analogy which suggested it was false; and if the test failed, it would remain untrue, however complete and satisfactory the analogy appeared to be.

A theory is not a law. But a theory is not a law; it cannot be proved, as a law can, by direct experiment; and the method by which it was suggested is not unimportant. For a theory may often be accepted without the performance of any additional experiments at all; so far as it is based on experiments, those experiments are often made and known before the theory is suggested. Boyle's Law and Gay-Lussac's Law were known before the dynamical theory of gases was framed; and the theory was accepted, or partially accepted, before any other experimental laws which can be deduced from it were known. The theory was an addition to scientific knowledge which followed on no increase of experimental knowledge and on the establishment of no new laws; it cannot therefore have required for its proof new experimental knowledge. The reasons why it was accepted as providing something valuable which was not contained in Boyle's and Gay-Lussac's Laws were not experimental. The reason for which it was accepted was based directly on the analogy by which it was suggested; with a failure of the analogy, all reason for accepting it would have disappeared.

The conclusion that a theory is not a law is most obvious when it is such that there are hypothetical ideas contained in it which are not completely determined by experiment, such ideas for example as the m, n, x, y, z in the dynamical theory of gases in its simple form. For in this case the theory states something, namely propositions about these ideas separately, which cannot be either proved or disproved by experiment; it states something, that is, which cannot possibly be a law, for all laws, though they may not always be capable of being proved by experiment, are always capable of being disproved by it. It may be suggested that it is only because the theory which has been taken as an example is of this type that it has been possible to maintain that it is not a law. In the other extreme, when all the hypothetical ideas are directly stated by the dictionary to "be" measurable concepts, the conclusion is much less obvious; for then a statement can be made about each of the hypothetical ideas which, if it is not actually a law, can be proved and disproved by experiment. This condition is attained only in theories of a special, though a very important type, which will receive attention presently.

The case which demands further consideration immediately is that in which the dictionary relates functions of some, but not all, of the hypothetical ideas to measurable concepts, and yet these functions are sufficiently numerous to determine all the hypothetical ideas. In this case it is true that propositions can be stated about each of the hypothetical ideas which can be proved or disproved by experiment. Thus, in our example, if one litre of gas has a volume mass of 0·09 gm. when the pressure is a million dynes per cm.^{2} then, in virtue of this experimental knowledge, it can be stated that v is $1{\cdot}8 \times 10^5$ cm./sec. A definite statement can be made about the hypothetical idea v on purely experimental grounds. If the dictionary mentioned sufficient functions of the other ideas, similar definite experimental statements might be made about them. If the theory can thus be reduced to a series of definite statements on experimental grounds, ought it not to be regarded as a law, or at least as a proposition as definitely experimental as a law?

I maintain not. A proposition or set of propositions is not the same thing as another set to which they are logically equivalent and which are implied by them. They may differ in meaning. By the meaning of a proposition I mean (the repetition of the word is useful) the ideas which are called to mind when it is asserted. A theory may be logically equivalent to a set of experimental statements, but it means something perfectly different; and it is its meaning which is important rather than its logical equivalence. If logical equivalence were all that mattered, the absurd theory of p. 123 would be as important as any other; it is absurd because it means nothing, evokes no ideas, apart from the laws which it explains. A theory is valuable, and is a theory in any sense important for science, only if it evokes ideas which are not contained in the laws which it explains. The evocation of these ideas is even more valuable than the logical equivalence to the laws. Theories are often accepted and valued greatly, by part of the scientific world at least, even if it is known that they are not quite true and are not strictly equivalent to any experimental laws, simply because the ideas which they bring to mind are intrinsically valuable. It is because men differ about intrinsic values that it has been necessary to insert the proviso, "by part of the scientific world at least"; for ideas which may be intrinsically valuable to some people may not be so to others. It is here that theories differ fundamentally from laws. Laws mean nothing but what they assert. They assert that certain judgements of the external world are related by uniformity, and they mean nothing more; if it is shown that there may be a case in which these judgements are not so related, then what the law asserts is false, and, since nothing remains of the law but this false assertion, the law has no further value. We can get agreement concerning this relation and we can therefore get agreement as to the value of laws.

The development of theories. The distinction between what a theory means and what it asserts is of the utmost importance for the comprehension of all physical science. And it is in order to insist on this distinction that the case has been considered when all the hypothetical ideas can be determined by experiment, although not all of them are stated by the dictionary to "be" concepts. As a matter of fact I do not think this case ever occurs, though we cannot be certain of that conclusion until all physics has been examined in detail. There is always, or almost always, some hypothetical idea propositions concerning which cannot be proved or disproved by experiment; and a theory always asserts, as well as means, something which cannot be interpreted in terms of experiment. Nevertheless it is true that a theory is the more satisfactory the more completely the hypothetical ideas in it can be experimentally determined; those ideas may be valuable even if nothing can be stated definitely about them, but they are still more valuable, if something can be stated definitely. Thus, in our example, the theory is valuable even though we cannot determine m or n; but it will be more valuable if they can be determined. Accordingly when a theory containing such undetermined ideas is presented and appears to be true, efforts are always directed to determine as many as possible of the undetermined ideas still remaining in it.

The determination of the hypothetical ideas is effected, as we have noticed

before, by the addition of new propositions to the hypothesis or to the dictionary, stating new relations of the hypothetical ideas to each other or to the concepts. The process demands some attention because it is intimately connected with a very important property of theories, namely their power to predict laws in much the same way as laws predict events. In passing it may be noted that a failure to distinguish a law from an event and a consequent confusion of two perfectly distinct kinds of prediction has also tended to obscure the difference between a theory and a law.

There is an important difference between the addition of new propositions to the hypothesis and to the dictionary. The hypothesis gives the real meaning of the theory and involves the analogy which confers on it its value; the dictionary uses the analogy, and the propositions contained in it are usually suggested by the analogy, but it adds nothing to it. Accordingly a change in the hypothesis involves to some extent a change in the essence of a theory and makes it in some degree a new theory; an addition to the dictionary does not involve such a change. If, then, a new law can be deduced by the theory by a simple addition to the dictionary, that law has been in the fullest and most complete sense predicted by the theory; for it is a result obtained by no alteration of the essence of the theory whatever. On the other hand if, in order to explain some new law or in order to predict a new one, a change in the hypothesis is necessary, it is shown that the original theory was not quite complete and satisfactory. The explanation of a new law and the determination of one more hypothetical idea by addition to the dictionary is thus a very powerful and convincing confirmation of the theory; a similar result by an addition to the hypothesis is, in general, rather evidence against its original form.

But the degree in which the necessity for an alteration in the hypothesis militates against the acceptance of the theory depends largely on the nature of the alteration. If it arises directly and immediately out of the analogy on which the hypothesis is based, it scarcely is an alteration. Thus, in the theory of gases in the form in which it has been stated so far, the only dynamical proposition (or more accurately the only proposition analogous to a dynamical law) which has been introduced is that the momentum is reversed in sign at an impact with the wall, while its magnitude is unchanged. But in dynamical systems this condition is fulfilled only if the systems are conservative; it is natural therefore to extend the hypothesis and to include in it any other propositions concerning the hypothetical ideas which are analogous to other laws[1] of a conservative system. Such an extension involves no essential alteration of the theory, but it permits the explanation of additional laws and thus provides arguments for rather than against the theory. For example, if the extension is made (the new propositions are so complex if they are stated in a full analytical form that space need not be wasted in stating them) the effect on the behaviour of the gas of the motion of the walls of the vessel

[1] The "laws" of a conservative system are not really laws, but for the present they may pass as such.