cism of Christianity, in *Samidare-shō,* focuses on the idea that a foreign religion that puts God before devotion to one's lord and one's father cannot be tolerated.

Bibliography

Miura's works are available in Japanese in *Baien zenshū* ("The Collected Works of Miura Baien"), 2 vols. (Tokyo, 1912). See also G. K. Piovesana, "Miura Baien, and his Dialectical and Political Ideas," in *Monumenta Nipponica,* Vol. 20 (1965), 389–443, which contains a translation of Miura's letter "Answer to Taga Bokkyō." See also W. T. de Bary, Ryusaku Tsunoda, and Donald Keene, eds., *Sources of Japanese Tradition* (New York, 1958), pp. 489–497; N. S. Smith, "An Introduction to Some Japanese Economic Writings of the 18th Century," in *Transactions of the Asiatic Society of Japan,* 2d series, Vol. 11 (1934), 80–88; L. Hurvitz, "The *Samidare-shō,*" in *Monumenta Nipponica,* Vol. 8 (1953), 289–326; Vol. 9 (1953), 330–356.

GINO K. PIOVESANA, S.J.

MODAL LOGIC. See LOGIC, MODAL.

MODELS AND ANALOGY IN SCIENCE. The term "model" has become fashionable in the literature and philosophy of science, with the result that the many different senses of the term need to be distinguished before the philosophical problems connected with models in the sciences can be understood. This article will begin with a classification of some of the more important senses of "model" before discussing the relevant philosophical issues.

Logical models. Formal logic is concerned with sets of axioms and their deductive consequences and also with the interpretations of these axioms and theorems in "models"—that is, sets of entities that satisfy the axioms. These relationships are most easily exemplified in terms of elementary geometry. Suppose a formalized geometry contains as an axiom the sentence "Any two points lie on one and only one straight line." In a fully formalized system there will be no definition of the terms "point" and "straight line" apart from this axiom and others in which these terms appear. As far as the formal system is concerned, the use of these terms is wholly defined by their relationships as given in the axioms and their deductive consequences. If we ask what the axioms are about, the only answer that can be given is that they are about just those sets of entities that satisfy the axioms.

One such set of entities clearly consists of the points and straight lines drawn in geometrical diagrams, or, rather, idealizations of these, which accurately reproduce the relationships specified in the axioms. However, it does not follow that this obvious interpretation of the axioms is the only possible one. In the geometric example the axiom may otherwise be interpreted in terms of certain sets and their members, so that the axiom would read "Any two individuals are comembers of one and only one set." Similarly, a formalized Boolean algebra can be interpreted as a calculus of classes, as a calculus of propositions, or in terms of spatial areas as in the Venn diagrams. Any set of entities that constitutes an interpretation of all the axioms and theorems of a system and in which those axioms and theorems hold true is called a model (in the logician's sense) of that system.

Such an informal characterization of this sense of "model" is, of course, entirely inadequate for the logician's purposes. But it is sufficient to indicate, first, how the term "model" has become attached to certain semiformal and nonformal systems in the empirical sciences and, second, what crucial differences exist between these logical models and those that are of interest in science. Some uses of "model" in science, such as nineteenth-century mechanical "models" of the ether, antedate the logician's use; others are consequences of it. It can be said as a preliminary that most uses of "model" in science do carry over from logic the idea of interpretation of a deductive system. Most writers on models in the sciences agree, however, that there is little else in common between the scientist's and the logician's use of the term, either in the nature of the entities referred to or in the purposes for which they are used.

Replicas and analogue machines. There is a sense of "model" in science that is both nearest to its sense in ordinary language and furthest from the logician's sense. Replicas, scale models, and analogues are familiar in various contexts and may be said to provide, after logical models, the second main source of ideas associated with the term "model" in the sciences. They may be used in science for expository purposes or even as calculating devices in cases where the building of a replica or analogue of a system as a working model is the simplest method of investigating the consequences of those natural laws that the system is believed to satisfy. Various examples include wind-tunnel experiments, crystallographic models, electronic models of nerve nets, and hydraulic models of economic supply and demand. Not all these are examples of straightforward replicas, however; some of them do not resemble in substance the thing modeled but are merely similar in certain of the relations between its parts. It is perhaps better, therefore, to call them analogue machines. Thus, economic supply and demand does not consist of pipes carrying colored fluids, but the relations exhibited by such a model may enable conclusions to be drawn in an economic system when the appropriate interpretations are made. In such cases the similarity of relations between model and system modeled may be called isomorphism. This relation of isomorphism may be linked with the concept of logical model by remarking that if the laws of the system were explicitly set out in a formal system, then model and thing modeled would both be models of that system in something like the logical sense. The relation of isomorphism between the model and the thing modeled would therefore be the relation between two interpretations of the same formal system. But this way of looking at analogue machines may be highly artificial in many practical cases, because such machines are often constructed precisely for cases where there is no known mathematical specification of a system or where this specification is so complex that the explicit drawing of deductive consequences is impossible or impracticable. Where this is the case, it is dangerous to attempt to apply to scientific models those arguments that are valid in connection with logical models of formal systems.

Before describing the senses of "model" that are more central to the structure of theoretical science and that contain some of the features of both logical and replica models, it is useful to give some account of the associated notion of analogy in terms of which different kinds of models can best be categorized.

Analogy in science. The relation between model and thing modeled can be said generally to be a relation of analogy. Two kinds of analogy relation can be distinguished in connection with models in the sciences. First, in the case of a logical model of a formal system, there is analogy of structure or isomorphism between model and system, deriving from the fact that the same formal axiomatic and deductive relations connect individuals and predicates of both the system and its model. This isomorphism consists of the correspondence between individuals and predicates of the system and the terms that are their interpretations in the model. Derivatively, we may say that there is an analogy of the same kind between two different models of the same formal system. A swinging pendulum and an oscillating electric circuit, for example, are analogous by virtue of the formal relations described in a wave equation satisfied by both. Let us call this type of analogy between systems formal analogy.

Second, however, we must consider the analogy exhibited by a replica with its parent system, which consists in something more than formal analogy. In a formal analogy there may be no similarity between the individuals and predicates of two models of the same formal system other than their relation of isomorphism. But in a replica model there are also what might be called material similarities between the parent system and its replica. The wings of an aircraft and its replica, for example, may have similar shape and hardness and may be made of the same material although they differ in at least one respect, size. Where two systems exhibit such similarities, which are not—or not simply—similarities by virtue of being logical models of the same formal system, we shall say that they have material analogy.

Two systems may have formal but not material analogy; two examples are the hydraulic model of economic systems and the various mechanical and electrical models of the wave equation. The systems may have both kinds of analogy, as do such replicas and near replicas as crystallographic models. It does not seem possible to conceive of a material analogy without some formal analogy; if there is material analogy, there is presumably some consequent structural similarity that could—at least in principle—be formalized. The distinction between formal and material analogy is difficult to make precise, but its significance will become clearer in considering the functions of theoretical models.

The relation of analogy, whether formal or material, generally implies differences as well as similarities. In analogous systems let us denote the set of similarities by the term "positive analogy" and the set of differences by "negative analogy."

The types of models to be described with the aid of these concepts of analogy are classified mainly in accordance with their function in relation to theories rather than with their intrinsic character. Thus, a model having various functions on the same or different occasions may very often come under different categories, and models of very different intrinsic kinds may come under the same category. Many other types of classification of models can be and have been produced (depending on whether they are mechanical or electrical, micromodels or macromodels, and so on). The categorization to be given now, however, seems to raise the most interesting philosophical questions in regard to the functions of models in science.

Semiformal or mathematical models. The logical sense of "model" has led to widespread use of the word in connection with a variety of mathematical theories developed in the sciences. No element of a replica is involved in these theories, and their interpretation is in terms of mathematical concepts such as probability or the elements of a geometry. It has become common to speak of "probabilistic models" of, for example, psychological learning theory or population dynamics. In this context "model" refers to a mathematical theory containing the axioms of probability together with an interpretation of all or some of the nonlogical constants and variables of the theory into empirical observables. In these cases use of the word "model" seems to borrow most of its appropriateness from the logical sense, and the analogy involved is almost wholly formal. Insofar as it is merely formal, some writers, such as Max Black, have denied that it has any causal or explanatory force, since the theories involved are no more than convenient mathematical expressions of the empirical data. In some cases, however, these theories do seem to have some element of material as well as formal analogy. For example, the "probability" that is a limiting-frequency interpretation of the axioms of probability exhibits some material analogy with the logical or range model of probability, for the similarity or correlation of the two notions in, for instance, games of chance exists not only by virtue of formal analogy with the same axiom system but also might well be apprehended independently of knowledge of that system. Where such material analogy does exist in connection with theoretical models, we may well say that a mathematical model does have causal, predictive, and explanatory force as an interpretation of a formal system.

Simplifying models. The term "model" is sometimes used to denote systems that deliberately simplify and even falsify the empirical situation under investigation for purposes of convenience in research or application. Such idealizations as ideal gases come into this category, as do such simplifying statistical approximations as "smoothed-out universes" in cosmology. It is also convenient to include in this category archaic models, which have been developed in now falsified theories but which still have some use as convenient approximations in applied rather than pure science. Examples are the model of heat as a fluid or of faculty psychology, in which man is seen as a nexus of interacting faculties of reason, will, and emotion. Archaic models are those that have a large and deliberate element of negative analogy with the relevant empirical system, in respects sufficiently important to have led to abandonment of these models in connection with current theories. But insofar as they are still at all useful, they must retain sufficient positive analogy in other respects to enable some correct conclusions to be drawn from the comparison of system and model.

Theoretical models. There are kinds of models, which we shall conveniently group as theoretical models, that are much more intimately associated with the structure of theories than simplifying models. Roughly speaking, these are models that appear—at least at first sight—to be identical with the relevant theory, as may be indicated by means of some examples. The explanation of light phenomena in terms of light corpuscles may be spoken of indifferently as a corpuscular model or a corpuscular theory of light. Similarly, the model of the DNA molecule, in at least one sense of "model," is identical with the theory of the molecular structure of DNA. We speak of Bohr's model of the atom, referring to the theory that was proposed to account for certain quantum phenomena. In cosmology we refer to "world models," which are theories of the structure of the universe. If there is any difference in such cases between the uses of the terms "model" and "theory," it is probably connected with the degree of acceptability of the theory. Thus, Bohr's theory, which is a rather radical departure from previous physics, or a theory of light that is not fully established and to which there are viable alternatives, may be called a model. It would, however, be odd to speak today of a wave model of sound, since a theory of sound in terms of wave motion is fully established and is even regarded as factual rather than theoretical.

The question of the degree to which a theory is accepted does not, however, seem to be the most important consideration in leading some philosophers to maintain a distinction between theoretical models and the theories of which they are models. To understand their motives, let us try to abstract from the previous examples the salient features of theoretical models. First, these frequently are models in something like the logical sense of being interpretations of a formal or semiformal theoretical system from which the phenomena are deducible. Thus, a system of mechanical corpuscles is an interpretation of Newton's laws of motion, and in this interpretation the linear propagation and reflection properties of light can be deduced from Newton's laws. If the Bohr model of the atom had turned out to be acceptable in a more developed quantum theory, it also would have been an interpretation of the formal structure of quantum theory. In the case of the DNA molecule, it is not so clear that there is any formal theory of which it is a model; the presumption is that a wave mechanics adequate to describe such complex structures as organic molecules would be such a theory. This example indicates that in science, unlike logic, the notion of model is not dependent on prior development of a formal theory.

The second notable feature of theoretical models is that they are dependent on some system or, if it is known, on the theory of the system, which is epistemologically prior to and independent of the particular phenomena that the model is invoked to explain. In other words, models of this kind provide explanation in terms of something already familiar and intelligible. This is true of all attempts to reduce relatively obscure phenomena to more familiar mechanisms or to picturable nonmechanical systems (such as Bohr's atom), and it is true of geometrical models of the expanding universe and in topological models of brain structure, to give only a few examples. Basically, the theoretical model exploits some other system (such as a mechanism or a familiar mathematical or empirical theory from another domain) that is already well known and understood in order to explain the less well-established system under investigation. This latter may be called the explanandum. What chiefly distinguishes theoretical models from other kinds is a feature that follows from their associating another system with the explanandum. This is that the theoretical model carries with it what has been called "open texture," or "surplus meaning," derived from the familiar system. The theoretical model conveys associations and implications that are not completely specifiable and that may be transferred by analogy to the explanandum; further developments and modifications of the explanatory theory may therefore be suggested by the theoretical model. Because the theoretical model is richer than the explanandum, it imports concepts and conceptual relations not present in the empirical data alone.

The last point can be used to characterize the difference between theoretical models and models in other categories. Almost any model or interpretation carries some surplus meaning. If, however, a model is used in a way that exploits this surplus meaning in prediction and explanation, we shall call it a theoretical model. Here another distinction must be made. Any model derived in an unsophisticated way from a familiar system (such as a mechanism) inevitably has negative as well as positive analogies with the explanandum. When a billiard ball model of gases is proposed, it is not intended that every feature of billiard balls—for example, their size or color—should be ascribed to gases. There is always a negative analogy that is implicitly recognized and tacitly ignored. We can therefore make a distinction between the model as exhibited by the familiar system and the model as it is used in connection with the theory. The latter is a conceptual entity arrived at by stripping away the negative analogy, and it is only this that can plausibly be identified with the theory. It is only this that we shall in future speak of as the theoretical model proper.

The sense in which the theoretical model and the theory can be identified can now be brought out in the following way. The model is first proposed because there is some obvious positive analogy (usually material as well as formal) between it and the explanandum. But the theoretical model that results from ignoring the negative analogy has more than simply a remaining positive analogy with the explanandum. If this were not so, the theoretical model would be identical with the explanandum and not richer, as we have required. In addition to the known positive analogy, there is a set of properties of the model whose positive or negative analogy is not yet known. Let us call this set the neutral analogy. Exploitation of the model consists in investigating this neutral analogy and in allowing the neutral analogy to suggest modifications and developments of the theory that can be confirmed or refuted by subsequent empirical tests.

Function. Few philosophers would deny that theoretical models may have the heuristic function in relation to theories that has just been described. The main philosophical debate about models concerns the question of whether there is any essential and objective dependence between an explanatory theory and its model that goes beyond a dispensable and possibly subjective method of discovery

The debate is an aspect of an old controversy between the positivist and realist interpretations of scientific theory. Many episodes in the history of science may be regarded as chapters in this controversy, including application of Ockham's razor to scientific theories, the Newtonian–Cartesian controversy over the mechanical character of gravitation, nineteenth-century debates about the mechanical ether and the existence of atoms, and Machian positivism. In its modern form the argument for the essential dependence of theories on models was first developed in 1920 by N. R. Campbell in *Physics, the Elements*, and it is convenient to state the argument mainly in his terms.

Campbell's interpretation. Campbell attacked the contemporary positivist view, expressed by Heinrich Hertz, Ernst Mach, Pierre Duhem, and others, that models are merely dispensable aids to theory construction and can be detached and discarded when the theory is fully developed. In *Physics* Campbell first sets out explicitly the structure of a particular theory (the elementary kinetic theory of gases) in what later came to be called the hypothetico-deductive form. This form exhibits theories as made up of three elements—a formal deductive system (hypothesis) of axioms and theorems; a "dictionary" for translating some of the terms of the formal system into experimental terms; and experimental laws such as, in this example, the Boyle and Charles gas laws, which are confirmed by empirical tests and also can be deduced from the system of hypothesis plus dictionary. This structure is roughly what positivists and formalists regard as the essence of an explanatory theory. As distinct from various views demanding explicit definability in empirical terms of every nonlogical concept in a theory, Campbell points out that the hypothetico-deductive structure does allow for "hypothetical ideas," the interpretations of which were later called theoretical concepts. These concepts appear in the formal theory as part of the machinery of deduction; they do not appear in the theory's dictionary and are therefore given no explicit empirical interpretation. Further, Campbell argues that the hypothetico-deductive form is insufficient to account for an explanatory theory as understood in science. There is, he maintains, an essential fourth element in theories—namely, the analogy, which is exemplified in gas theory by the model of point particles moving at random in the vessel containing the gas. In this model all the theoretical concepts such as molecule, as well as the position, velocity, and mass of molecules receive an interpretation in particle mechanics, although they are not directly observable.

Campbell has two main arguments for his view that the particle model is essential to the structure of the theory of gases. First, it is intellectually satisfying as an explanation of the empirical data. Here Campbell implicitly contradicts the later analysts of explanation who regard the hypothetico-deductive structure itself as sufficiently explanatory and reject further criteria, such as familiarity or intellectual satisfaction, for explanation. Second, and more cogently, Campbell draws attention to the dynamic character of theories and their use in prediction. Using the particle model, he shows that it allows modifications and extensions of the theory that issue in empirical predictions over a wider domain of phenomena than those initially explained. Furthermore, his arguments implicitly demand a model that, in the above terminology, has not only formal but also material analogy with the explanandum; such would be a vessel containing a gas (for example, a balloon) as model for an elastic Newtonian particle. Predictivity is one of the characteristics demanded of satisfactory scientific theories, and Campbell argues that without material analogy there are no rational, nonarbitrary grounds for prediction. His argument has been extended and developed (by E. H. Hutten, M. B. Hesse, R. Harré, and others); it has also been subject to various objections, and formalist alternatives have been proposed by R. B. Braithwaite and others. Three distinct problems can be distinguished in these subsequent discussions—the predictivity of theories, the meaning of theoretical concepts, and the question of the realistic interpretation of models.

Predictivity. Most disputants are agreed that predictivity in some sense is a requirement for satisfactory explanatory theories, but some deny that theoretical models are essential or even helpful for the satisfaction of this requirement. First, it is argued that even if there is such a model for a given theory, no argument by analogy with the model guarantees the truth of the predictions thereby derived. This must be conceded, for no empirical predictions can be guaranteed, however derived. But the objection presupposes that there are no grounds for holding that arguments by analogy have at least some inductive force and that these grounds may be stronger than for other methods of making predictions from theories. The truth of this presupposition is by no means obvious (see P. Achinstein, "Variety and Analogy in Confirmation Theory," and M. B. Hesse, "Analogy and Confirmation Theory").

Second, it is argued that theories without theoretical models may use criteria other than analogy for the purposes of extension and prediction. Such formal characteristics as simplicity, symmetry, or mathematical elegance may be exploited to modify or extend the theory and thus to derive from it new consequences that can be empirically tested. Introduction, in the interests of symmetry, of the displacement-current term in Maxwell's equations is cited as an example of this process. Furthermore, although it is pointed out that in modern physics it has been shown conclusively that no models of the classical type are possible, the quantum theory does seem to have all the required characteristics, including predictivity, even though no other type of model has been introduced. Further discussion of this point demands a closer analysis of what is meant by predictivity and also of the assumption that quantum theory, where it is predictive, does not still make essential use of models in some sense.

The meaning of theoretical concepts. In the hypothetico-deductive scheme theoretical concepts are not given explicit meaning in terms of observables. Thus, there is the question of how theoretical concepts are meaningful or, expressed more precisely, if the rules of the formal hypothesis give the syntax of the system, what gives its semantics. In the Campbellian tradition the answer has been that the semantics is given by the model, which is intelligible independently of the explanandum; hence, the model contributes to the meaning of the theoretical concepts an element not derived from any direct connection with the observable explanandum. Those who wish to relegate models to mere heuristic devices argue, on the other hand, that no such nonempirical element in the meaning of theoretical concepts is required. An extreme version of this formalist view would hold that no interpretation at all of the theoretical terms is required, that the theory can, in fact, be viewed as a black box into which data are fed and out of which predictions emerge without any question arising as to the meaning of the intervening deductive machinery or its axioms. Thus, such theoretical terms as "electron" and "electromagnetic field" are nothing more than arbitrary names for certain parts of the deductive machinery and must be entirely divested of any associations with a model. A somewhat less extreme view, which Braithwaite labeled contextualism, holds that theoretical concepts have meaning that is wholly derived from the empirical consequences which can be drawn from the theory; meaning in this sense must be regarded as implicit or contextual, in contrast to the explicit empirical meaning of observables. Thus, in this view "electron" means just that entity that has the relations to the other entities of atomic physics that are specified in the formal system of physics; this formal system is such that empirically confirmed relations between observables can be deduced from it, although "electron" does not appear explicitly among these observables. In this view the interpretation of theoretical terms clearly has some of the features of an interpretation into a logical model but has no reference to the further, familiar system required for theoretical models. Whether such an analysis of "meaning" sufficiently accounts for the use of theoretical concepts remains controversial. It must also be said that the notion of "contextual meaning" has by no means been fully worked out.

A different kind of denial of the relevance of the Campbell account comes from those who hold that there is no problem of the meaning of theoretical concepts because these are learned like any other linguistic terms by their use in the development of theoretical science. Theoretical terms in this view are in some way extensions of the language about observables used to refer to nonobservables. The potentiality for such extensions is said to be always present in language—for instance, even in as childish an example as "people too little to see." Again, however, no adequate account of these linguistic extensions has been given, and in particular it has not been shown that they are independent of exactly the kind of analogical meaning that Campbell began to analyze.

Realism. It might be agreed that models are essential for prediction and for giving semantic interpretation to theories, although one can still deny that they are intrinsically part of the theory in the sense of being its real reference. Campbell himself was in fact nearer to the formalists than the realists on this point, for in answer to the question "Are there molecules?" he denied that the particle model implied the existence of molecules as real constituents of gases. According to him, if we answer the question affirmatively, all we intend is a shorthand assertion of all that has been said about the essential function of the analogy between the gas theory and particle systems. Contextualists, on the other hand, in answering the question affirmatively, intend only to assert the existence of entities that, by means of their relations to other entities specified by the theory, issue in empirically confirmed laws as deductive consequences.

It would seem more natural to hold, as was naively held by almost all theorists before the nineteenth century, that when a theory is developed in terms of a model, the model is the description of the way the world is conceived by that theory. That is, gases are really made up of molecules, light is really transmitted by wave motion in the ether, and so on. In other words, the model containing the positive and neutral analogy with the explanandum (and not the negative analogy) is identifiable with the theory of the explanandum, and this theory has a real reference to the domain of the explanandum. There are several reasons why some writers wish to deny such an identification. First, it is considered dangerous to identify or confuse model with theory, because the model may have implications that turn out to be untrue of the explanandum. This is a weak objection, because it can be made against any theory having implications beyond what has been directly confirmed—that is, against any theory with predictive potentialities. It also overlooks the possibility of conceiving a model from which the negative analogy has been excluded. If there are implications that are known not to be true and if these are not central to the model, they can be deliberately ignored and are therefore not dangerous.

Second, it is held that models are used in situations where deliberate simplification and distortion are intended and that therefore they cannot be identified with the theory of which they are imperfect interpretations. Some models are undeniably used in this way, as has been described, but it does not follow that all are. In some cases it is even possible to show how initially distorting models have been modified and refined so as to become consistent with theory and explanandum, to become, in the phrase of R. Harré, "candidates for reality."

A third argument is that even if models are accepted as essential ingredients of theories, there is no evidence (other than their functions in relation to prediction and meaning) for endowing them with "reality." This objection may be associated with the stronger presupposition that only that which is in some way directly observed can be real. But in both versions there is still the question of what is involved in ascribing physical reality to entities and what is the relation between the theoretical and observational in science. These problems are dealt with in LAWS AND THEORIES.

Bibliography

Achinstein, P., "Variety and Analogy in Confirmation Theory." *Philosophy of Science*, Vol. 30 (1963), 207–221.

Beament, J. W. L., ed., "Models and Analogues in Biology." *Symposium of the Society for Experimental Biology.* Cambridge, 1960. Discussions of various kinds of model in biology.

Black, M., *Models and Metaphors.* Ithaca, N.Y., 1962. See Chs. 3 and 13 for an important analysis of linguistic metaphor and application to theoretical models.

Braithwaite, R. B., *Scientific Explanation.* Cambridge, 1953. Chs. 1–4.

Braithwaite, R. B., "Models in the Empirical Sciences," in E. Nagel et al., eds., *Proceedings of the Congress of the Interna-*